RESOURCE-BASED SUSTAINABLE DEVELOPMENT: AN ENERGY PLANNING MODEL FOR NIGERIA

By

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Abstract

In this paper, a stylized optimization model is presented for Nigeria's energy sector that incorporates important socio-economic objectives with traditional energy-sector planning goals of resource allocation and cost minimization. With a rapidly growing economy, Nigeria aspires to use revenue from rent of its natural resources to fund economic development. To this end, the formulation of energy-sector planning techniques has been an important objective for scholars and policy makers. We offer a new approach with our development of a resource-planning tool useful for a peculiar economic environment that integrates elements from the macroeconomics, open economy and natural resource literatures. Our model for economic development in Nigeria characterizes the interdependent intertemporal optimization behavior of major players in the economy—the government, private sector (households and firms), public utilities and foreign oil companies. The model, which is based on established results from the literatures on the optimal exploration for and extraction of exhaustible resources, is employed in a deterministic simulation exercise that takes the form of a dynamic policy game.

Keywords

Optimization * Extractive * Energy * Intertemporal * Dynamic * Policy * Economic Planning * Development * Representative Agent * Natural Resources

Introduction

Nigeria aspires to use revenue from its rents from natural resource extraction, particularly of petroleum resources, to fund social and economic development. The government goals for economic empowerment include the development of material and human capacity so as to enhance sustainability and reduce the current heavy dependence of the economy on a single nonrenewable resource with highly volatile output prices. To achieve these goals, urgent attention has been drawn to other sectors of the economy with potential for significant impact on the general economy, with the energy sector identified as one of such sectors. (See e.g., World Bank, 2002.) Energy

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planning and development, particularly of electricity generation and distribution are important considerations in the discussion on economic development in Nigeria. While a number of tools have been applied to short to long-term economic planning for the country that emphasize energy development, there may be a need for updating of the modeling tools is use. This is particularly in light of recent structural changes in the economy and of emerging developments in the theory and methods of economic and energy planning and modeling. New methods that we develop for macroeconomic planning account explicitly for the oil sector as an important player in the economy, tracking the implications of sector-wide changes in this industry on other sectors. The methods also account for the increasing focus of the government on such objectives as human capital development, the expansion of a nonoil sector with export potential and increased access to energy for residential and industrial consumers.

In this paper, we integrate elements from the microeconomic, open-economy macroeconomic and natural-resource economics literatures in the development of a stylized optimization model for the energy sector that incorporates public sector objectives for social and economic development with more traditional energy-sector planning goals of resource allocation and cost minimization. By characterizing activities in the real economy as resulting from the intertemporal optimization decisions of representative agents, our model is founded on microeconomic theory and is much in the spirit of macrodynamic models outlined in Turnovsky (2000). We incorporate sector economic activities in simulation experiments that take the form of dynamic games between the relevant agents. We set out to demonstrate the application of such a tool to the economy of Nigeria. To the best of our knowledge, no such approach has been applied to resource planning and economic development in Nigeria. While the results obtained from the simulation exercises only lend limited direct application to policy implementation, they provide a sense of the types of policy plans that will be required to implement a comprehensive and integrated national plan for Nigeria's energy sector, offering useful insights for the development of tools with greater application potential.

The next section provides a summary of the current state of the energy sector in Nigeria and it's planning, followed by a summary of the relevant literature. A characterization of major economic agents – the government, private sector households and businesses, multi-national oil companies, and public sector electricity companies - and their intertemporal optimizing behaviors is next presented. This is followed by a discussion on the data and methods of the model development and application to the empirical setting. We then describe a simulation exercise to determine the implications of relevant policy and agent decision variables. Finally, we present conclusions that can be drawn from this study.

Energy Sector and Planning in Nigeria

Many concerns have been raised regarding the capacity of the energy sector in Nigeria to support demand in an expanding economy. [3,4] For example, while urbanization rates have been on the rise, access to electricity remains severely limited with only about 40 percent of the (circa 140 million) population being connected to available electricity grids. Where customers are connected to the grid, frequent blackouts are the norm and many residential and commercial users go without, or rely on alternative sources of energy (ECN, 2003)[5]. These concerns are however not peculiar to the country as they echo reports of similar challenges faced by fast-growing economies in developing regions - China, India and South Africa being important examples. In general, local energy sector production has not kept pace with the expanded demand that has followed industrialization in these countries. For example, whilst the annual GDP growth rate in China was more than six times that recorded in the United States in 2007, per capita energy consumption in China trailed that of the United States by up to 80 percent in the same year. Similarly, electricity consumption in India was only 505 kilowatt hours per person in 2007 compared to more than 13,000 kilowatt hours per capita per annum in the United States in the same year. Meanwhile, GDP growth in India outstripped that of the U.S. by 4 percentage

[3] Energy demand here refers to transport, industrial and residential fuel and electricity use.
[4] E.g., various local and international media reports and a World Bank (2002) report.
[5] Alternative sources include diesel and petrol-powered generators for residential and industrial use.

points in the reported year. Relevant economic and energy indicators for selected countries are shown in Table 1. As shown in the table, while the rate of economic growth in Nigeria can be compared to that of say, South Africa, energy demand has remained considerably more modest.

Table 1: Economy and energy indicators for selected countries in 2007

	China	India	Nigeria	S. Africa
GDP annual growth rate (%)	13	9	6	5
GDP per capita (USD)	2,566	1,047	1,158	5,866
Energy consumption (million btu)	59	17	7	111
Electricity consumption per capita (kWh)	2,150	505	134	4,447
CO_2 emissions per capita (MT)	4.74	1.23	0.72	9.59
CO_2 emissions per GDP (MT/'000 USD)	2.20	2.44	5.39	2.16

Source: Authors' calculations based on World Bank and EIA data

Reports from recent surveys indicate that urban industries in Nigeria meet up to 70 percent of their energy needs (for heat and electricity) using privately-owned, customer-sited diesel and gasoline generators. The operating costs for this energy option tend to be up to five times more than what is charged by the publicly provided (but largely unavailable) grid electricity (Adenikinju, 2003). Further, up to 80 percent of households' cooking, heating and lighting energy needs are fulfilled using wood, kerosene and other alternative fuels (ECN, 2006). The uses of these alternative power sources could have serious economic as well as environmental implications.[6] Not unlike many other developing countries, environmental pollution in Nigeria may not seem to be a serious threat when measured using standard international measures such as total carbon dioxide (CO_2) emissions. Still, these levels could be considered high for the levels of economy output. For example, while the level of CO_2 emissions per capita in Nigeria was less than a unit metric ton in 2007, this level was more than twice (fourfold) that generated in South Africa (United States) on a national income

[6] Fuel wood use increases emissions of harmful air particulates while the use of alternative power generators by households and businesses increases noise and air pollution levels.

basis (see Table 1).[7] However, the need for increased productivity and rapid economic development cannot be over-emphasized and improved access and supply of electrical power has been touted as critical to sustaining industrial competitiveness and economic growth (see World Bank, 2002; and IMF, 2008).

One obvious objective for energy planning in Nigeria is the expansion of the capacity for grid electricity generation, transmission and distribution. Another goal is the diversification of the available energy resources. Up to 68 percent of the installed electricity generating capacity in Nigeria is in natural gas facilities while 31 percent of capacity is accounted for by hydroelectricity installations. Less than 1 percent of the installed capacity is for electricity generation from oil. However, the natural gas and hydro-facilities only account for, on average, 34 and 8 percent respectively, of the total energy produced in Nigeria. On the other hand, electricity generated from oil accounts for up to 58 percent of all energy consumed (EIA, 2007). The statistics may point out a need to improve capacity utilization in gas and hydro-electric facilities, or expand the capacity and efficiency of electricity generation from oil, or both.

In an obvious response to the underutilization issues in thermal energy production, the government in its role as a petroleum industry regulator has applied policies that discourage natural gas flaring by oil-producing companies.[8] Oil companies are instead encouraged to channel more of the natural gas produced in association with crude oil extraction to public energy utilities for electricity generation. There is also increased support in public and private stakeholder circles for the development of other alternative natural resources for electricity generation. It is anticipated that much of the investment required for development projects in the energy sector would come from crude oil rents paid to the federation accounts.

The overall planning challenge for the energy sector in Nigeria can be identified as the need to integrate goals for energy sector efficiency with more

[7] Productivity limitations rather than increased levels of particulate emissions from economic activity may drive these observations.
[8] While activities in the petroleum sector are often led by multi-national companies, the federal government takes on partnership, regulatory and supervisory roles.

economy-wide social and economic objectives.[9] Specifically, there is a need for a planning model that incorporates government targets for energy security, electricity production and consumption efficiency, and sectoral productivity growth with the objectives of businesses and households. We formulate within the framework of an intertemporal optimization model such a tool relevant for the Nigerian context.

Review of the Relevant Literature

Intertemporal optimization models have been useful for analyzing economic growth and other issues of macroeconomic policy. The theory of intertemporal planning has its foundations in work done by Ramsey (1928) on the optimal rates of savings and economic growth. Assuming that the welfare of the economy can be characterized by an aggregate utility function, and making assumptions of an infinite time horizon, no technological change, constant population and constant capacities for 'enjoyment' and 'sacrifice', and independence and aggregability of 'enjoyment' and 'sacrifice' over time, Ramsey formulated the now well-known mathematical rule for investments/savings in an economy, as in equation (1)[10]:

$$\partial k / \partial t = f((k,t), l(t)) - c(t) = [B - (U(c) - V(l)]/u(c) \quad (1)$$

where $\partial k / \partial t$ is the rate of change in capital, *k(t)*, over time, *t*; *c(t)* is defined as the total rate of consumption, and *l(t)* is the supply of labor. Income in the economy is represented by *f*, a general function of available labor and capital. *B* represents the maximum obtainable rate of utility; *U(c)* is the total rate of *utility* of consumption; *V(l)* is the total rate of *disutility* of the labor rate, *l(t)*; and *u(c)* is the change in total rate of utility as the consumption rate changes, $\partial U(c) / \partial c$. Ramsey's (1928) contributions have formed the basis for much of the later work.

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[9] Energy policy goals for Nigeria are outlined in ECN (2003). An abridged version is presented in the Appendix (3.A).

[10] In words, the rate of saving multiplied by the marginal utility of money is equal to the amount by which the total net rate of enjoyment of utility falls short of the maximum possible rate of enjoyment.

Other notable contributions have since been made to the literature on optimal savings and investments in an economy (see notes to Chapter 10 in Dasgupta and Heal (1979), and Part III of Turnovsky (2000) for an overview). For example, Hotelling in 1931 applied similar mathematical techniques as in Ramsey (1928) to problems of depletion of resources in an economy.

However, the earlier models tended to ignore *exhaustible* resources in the technical possibilities of the economies under investigation. Dasgupta and Heal (1974) addressed this problem in their work that explored the immediate consequences of incorporating the *existence* of exhaustible resources in intertemporal planning, investigating the determination of the optimal rates of depletion of exhaustible resources and of investments in the economy. A major contribution of that work lies in its proposition that the substitution between reproducible capital and exhaustible resources is an important determinant of the characteristics of an optimal policy.[11] By explicitly accounting for exhaustible resources in the long-term planning model, they more directly address planning issues that are critical to many resource-abundant (and often resource-dependent) economies. In another study, Solow (1974) showed that the inclusion of the exhaustible resources component in the intertemporal planning models led to interesting results but did not significantly alter the basic theoretical principles. Much of the more recent work on economic planning in resource-abundant developing countries builds on the important contributions made by Dasgupta and Heal (1974), and Solow (1974).

Hartwick (1977) developed a rule for investing rents from non-renewable resources. According to Hartwick's calculations, an economy could sustain a maximal constant level of consumption through successive generations by investing all of the profits or rents accrued from exhaustible resources in reproducible capital, and by investing only this amount. While this rule directly addresses ethical issues of intergenerational equity, it also has consequences for economic growth and development. Sachs and Warner (1997) report high natural resource dependence as one of three major structural conditions that dampen economic growth in Sub-Saharan

[11] This proposition lies at the heart of models of optimal long-term growth in resource-rich economies.

Africa and outline a need to better understand the growth experiences of the few successful resource-rich developing countries.

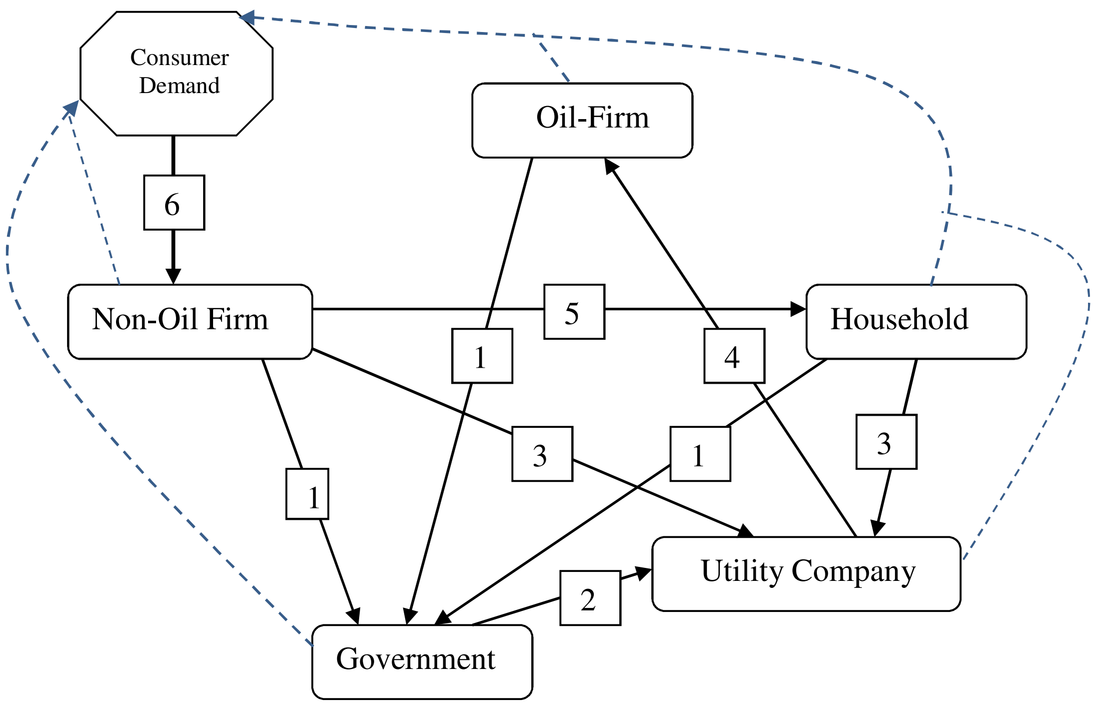
Hamilton, *et. al*., (2005) in a follow-up to the earlier work, tested the Hartwick (1977) rule using data on investments and rents from exhaustible resource extraction for 70 countries. They found that, in general, applying the standard rule as development policy would have been extreme. However, they show that resource-rich countries, by following an even moderate saving effort, could have substantially increased their wealth. Of particular importance to our paper are the results form Hamilton, *et. al*., (2005) that show that Nigeria could have been five times as well off as it was in 2005, had it followed the Hartwick rule in the 30 years prior; and that oil would have played a much smaller role in the economy with likely beneficial impacts on the nonoil sectors. The potential for oil-dependent economies to use oil rents for the development of other sectors in the context of an overall economic development framework remains of interest to researchers and planners alike. However, energy and economic planning in Nigeria have not gone without challenges.

Adamson (1978) pointed out energy planning in Nigeria in the 1960's and early 1970's largely ignored econometric forecasting owing mostly to issues of data unavailability and inaccuracy that rendered the models inapplicable. Iwayemi in 1978 proposed the application of a mixed-integer based programming approach to electricity allocation in Nigeria, making alternative assumptions on energy costs, and introducing a spatial dimension that accounted for the regional location of plants and the transmission of energy. This approach yielded an optimal generating mix from a set of fossil fuel and conventional hydro-plants that could meet demand projections over three decades. The proposed model provided important insights into the characterization of energy investment and supply in Nigeria. However, as pointed out by Adamson, while input-output and mathematical programming models generally fared better than econometric forecasting models, these models in themselves were not unaffected by the challenges posed by the paucity of accurate data in a peculiar planning environment. Other authors have proposed the use of multi-criteria programming methods for allocating energy resources among competing sectors of the economy. In particular, optimization models developed by the International Atomic Energy Agency (IAEA) have gained popularity in recent years (see ECN (2006)).

The earlier models for energy planning in Nigeria (e.g., Iwayemi, 1978) and the more recent modeling approaches (e.g., the IAEA models adopted for use in Nigeria), whilst providing important insights on the characterization of electricity demand and supply in Nigeria, may have considered the energy sector almost in isolation, largely ignoring important feedback with other sectors of the economy. The present research departs from the earlier work by integrating established tools from natural resource economics into a framework of intertemporally optimizing representative agents from the macroeconomics literature. Further, following dynamic game theory, the present research more completely characterizes the planning environment of interest by modeling the strategic pursuit of goals by several agents in the resource-rich economy.

Model Specification

Our formulation of the energy planning model represents a practical compromise between introducing sufficient detail to capture the stylized facts of the energy sector and economy in Nigeria and accommodating limitations that exist in the data. We specify the behavior of intertemporally optimizing representative agents in the manner presented in Turnovsky (2000). In line with a time-dependent macroeconomic system, the model is formulated in continuous time. Following the set-up of an open-loop noncooperative (Nash) differential game, we assume that each agent takes the optimizing behaviors of the other agents as given in making her own decisions. Figure 1 depicts the representative agents specified in our model of the economy. A government agent owns capital, receives oil rents from the activities of foreign-owned petroleum companies, and raises revenues from taxes on households and businesses. The government invests in the development of agricultural and human capital and is an investor in the energy sector, choosing its level of capital to fund electricity production. The petroleum sector is involved in oil and gas exploration and production and sells natural gas as energy inputs to a utility company while the utilities employ natural gas, labor and capital as inputs in electricity production. The electricity utility in Nigeria is currently undergoing a deregulatory process where the government-owned establishment maintains responsibility for electricity generation while offering opportunities to local private firms to bid for the rights to operate transmission and distribution activities. We ignore this level of detail in the current specification and assume that the utility entity operates all aspects of electricity supply to firms and households.



1. Taxes to government
2. Capital flows to public utilities
3. Payments to utility companies for energy outputs
4. Payments to oil producers for energy inputs
5. Labor wages to households
6. Sales of goods and services

Figure 1: Schematic representation of major agents in national economy showing income flows

The household agent makes decisions on her consumption of energy and other local and imported goods. She rents labor to private businesses, invests in corporate bonds and foreign assets and derives income from interests accrued on these assets. The representative agent for the private local firm uses labor, electricity and capital inputs to produce output in the economy. We assume a goods market composite of local and foreign consumer demand.

While the interaction of the agents is depicted within a macro environment, micro- and institutional foundations underlie the individual sectoral specifications. For example, microeconomic theory informs the characterization of the relevant producer and consumer decisions, the institutional structure of the Nigerian public sector informs our definition of the government agent behavioral function, and we incorporate the Pindyck (1978) model of the optimal exploration and production of nonrenewable resources in the natural resource management component of the economy. In developing the economy-wide modeling framework, we have found persuasive arguments by Wymer (1993, 1997) for specifying and estimating macro-dynamic models as nonlinear stochastic differential equation systems. Wymer (1993) argues that small, highly aggregated models of this type, based on sound theoretical foundations, can account for a broad range of macroeconomic activity while remaining amenable to mathematical and statistical analysis. In the spirit of this tradition, we assume that the relevant macroeconomic activities in Nigeria can be characterized by a stochastic nonlinear differential equation system representing the behavior of five agents.[12]

We next present the decision problems of the individual agents in some detail.

Government

The representative agent for the government chooses government consumption and investment in physical and human capital to minimize a weighted sum of deviations from policy targets for welfare and capital, subject to an intertemporal budget constraint. The objective functional and constraints are as in equations (2) through (9).

[12] The optimization problems of energy utilities and firms are given a static cast. This specification of these agents' decision behaviors is consistent with the two agents' not making choices affecting capital stocks; it also improves mathematical tractability of the model'.

$$\underset{\{I_{anr},I_{sc},I_o,OD_g\}}{Min} \int_{t0}^{\alpha} e^{-\beta gt} [\ 0.5\varpi_{g1}\left(K_{anr} - K_{anr_0} e^{\lambda 1t}\right)^2 + 0.5\varpi_{g2}\left(\Omega - \Omega_0 e^{\lambda 2t}\right)^2$$

$$+ 0.5\varpi_{g3}\left(E_o^{ng} - E_0^{ng} e^{\lambda 3t}\right)^2\ ]dt \qquad (2)$$

s.t.

$$\Phi_g orev + arev + \dot{B} = OD_g + rB + I_{anr}\left(1 + \frac{a_1 I_{anr}}{2K_{anr}}\right) + I_{sc}\left(1 + \frac{a_2 I_{sc}}{2K_{sc}}\right)$$
$$+ I_o\left(1 + \frac{a_3 I_o}{2K_o}\right) \qquad (3)$$

$$\dot{K}_{anr} = I_{anr} \qquad (4)$$

$$\dot{K}_{sc} = I_{sc} \qquad (5)$$

$$\dot{K}_o = I_o \qquad (6)$$

where

$$\Omega = (Ksc / Kf)^{\gamma w} \cdot (Y / pop)^{\alpha 1} \cdot ODg^{\alpha 2} \qquad (7)$$

$$Y = 0.5 \cdot \left(C + OD_g + I_f + I_{anr} + I_{sc} + I_o + orev - MGS + Y_f\right) \qquad (8)$$

$$arev = \tau_c\left(C + MGS\right) + \tau_y (r \cdot nfa) \qquad (9)$$

where the government agent chooses I_{anr}, I_{sc}, and I_o—her levels of investments in agriculture, social and community services, and other economic services—and OD_g, her levels of recurrent expenditure on goods and services.[13] K_{anr} is the accumulation of government capital in agriculture and can capture attempts to diversify the economy through investments in a non-oil sector with export potential. K_{sc} is capital accumulation in social and community services, a representation of development spending on human capital, and K_o is the capital accumulation in physical assets, including electricity generation capacity. Capital investments are assumed to be net of

[13] Education, health and housing make up community and social services provided by the government. Other economic services include energy, water resources, construction, and transport and communication.

depreciation and equations (4) to (6) define physical constraints on net accumulation of human and physical capital. K_{sc} and K_o enter into the government objective function through Ω and E^{ng}, endogenous levels of welfare and electricity power generation, respectively. Ω is a mathematical expression transforming (the proportions of) social and physical capital, the levels of per capita national income and the amount of government recurrent spending into a measurable index of social welfare. (See equation 7.) It can be directly affected by the government agent. On the other hand, E^{ng}, the value of electricity generated by the public utility, is not endogenous to the government agent's optimization problem as it is determined in the optimization problem of the utility agent (discussed below). However, government investment in the public utility, K_o, is important in electricity generation in the utility.

K_{anr0} is a specified target level of capital accumulation in agriculture, while Ω_0 and E_0^{ng} are corresponding target levels for welfare and energy development that are a function of government spending on social and other services and capital investment in the energy sector. As shown in equation (7), Ω is also influenced by the levels of other variables not directly controlled by the government agent. K_f is the level of investments that households make in private (non-oil) local firms. Y is national income, defined as the value of gross domestic production (GDP), while Y_f is the value of the net domestic output from the production function of the non-oil private firm (defined later for the local private firm), and *pop* denotes total population. The government seeks to minimize the period-to-period deviations from the target investment and social welfare levels.

In the intertemporal budget constraint specified in equation (3), the government expends income on capital (I_{anr}, I_{sc} and I_o) and recurrent (OD_g) projects. Costs of adjustments are imposed on period-to-period changes in the levels of physical stock and are assumed to be quadratic[14]. The adjustment costs serve two major purposes in the model. One, they impose a penalty on model solutions that represent

[14] The relevant quadratic function is $f_x(I_{x,} K_x) = I_x + a_x I_x^2 / 2K_x = I_x(1.0 + a_x I_x / 2K_x)$, where *x* represents the three forms of capital spending by the government, and *a* is the adjustment parameter associated with *x*.

substantial deviations in new investments from stable or long-term equilibrium path levels. Second, by introducing quadratic terms in investment costs, they ensure that the relevant control variables do not vanish from the equations that characterize an optimal solution to the agent's problem. (See an application in Donaghy *et al.,* 1999).

The government receives oil revenues, *orev,* from the multinationals; and taxes, *arev,* from households and non-oil firms. Oil revenues accruing to the federal government accounts are paid as a fixed proportion of the net proceeds from oil production (and exports), which are a function of the levels of oil production. Oil exploration and production in turn are set by the multinational (oil) company so that *orev* is taken as given in the set of equations for the government agent. Φ_g is the proportion of government income from oil revenues that is retained for government spending. This parameter has a value of one when there are no direct transfers of oil rents to citizens. Non-oil income (as shown in equation 9) includes taxes paid by households on local and foreign goods and on interests charged on assets held abroad. τ_c and τ_y are consumption and income taxes, respectively. The government also sells bonds, *B*, to raise government capital (Equation 3). *r* is the discount rate in the economy and the rate charged on holdings of bonds.

MGS is the sum of imports of goods and services while *Y* enters into the equations in the model as the sum of the economy's activities (equation 8). *C* is the sum of household expenditures on local goods and services. The value of aggregate exports from all sectors is a function of X_0, the value of crude oil exports.[15] On average, ninety-five percent of the oil produced in the country is exported and oil exports historically account for ninety-five percent of all exports (EIU, various issues). I_f is the level of new investments in local (non-oil) private businesses, determined by the household agent; and *DV* is the change in the level of inventoried goods. All value terms are in *naira*, the local currency.

The model parameters ϖ_{g1}, ϖ_{g2} and ϖ_{g3} are policy weights on the accumulation of the three types of capital. These should be determined empirically in the model, as

[15] Xo = P*Q, where P is defined as the real unit price of oil in Nigeria and is given by a price-dependent demand function, P = ($a - b \cdot Q$), where *a* and *b* are parameters of the price equation and *Q* is the quantity of oil produced, as defined for the oil firm agent.

should be *λ1, λ2,* and *λ3*, the time-dependent compounding factors assumed for the target capital accumulation, and γw, $\alpha 1$, and $\alpha 2$, the policy weights for the welfare variable (see equation 7) . The first-order necessary conditions for an optimal solution to the government agent intertemporal problem are presented in Appendix (B.1).

Multinational Oil Firm

Following Pindyck (1978), oil and gas firms are assumed to choose levels of exploratory and production activities. The firm seeks to maximize net revenues subject to cost relationships and restrictions on technology. We assume that the firm is able to augment its reserves at a rate that exceeds depletion of the resource in the short run. The aggregate output from the oil industry activities is exported or sold locally[16]. We specify the optimization problem of the firm as in equations (10) to (12):

$$\underset{\{W,Q\}}{\text{Max}} \int e^{-\beta t}\{(a-bQ)\cdot Q-(co/R)Q-(m+nW)\}dt \qquad (10)$$

s.t.

$$\dot{X}=r_o W^s e^{-uX} \qquad (11)$$

$$\dot{R}=\dot{X}-Q \qquad (12)$$

where Q is the level of production activity; W is the level of exploratory activity (wells drilled); R is the known level of reserves of the resource and X is the cumulative addition of known reserves. The first term in the objective function, i.e., $(a-bQ)\cdot Q$ in equation (3.10) is the sum of gross revenues received by the oil firm, while (co/R) and $(m+nW)$ are the extraction and discovery cost functions respectively. An average (annual) producer price of oil is endogenously determined for Nigeria and is specified in equation (10) as $(a-bQ)$.

[16] Non-export sales include supply of crude oil to local refineries and of oil and gas to power stations.

The change in the levels of cumulative additions to known reserves is defined in equation (11) as $r_o W^s e^{-uX}$, the discovery rate function. The discovery rate declines as exploration and discovery proceed over time; i.e., it becomes more and more difficult to make new discoveries (Pindyck, 1978). The parameters in the objective function, *a*, *b*, *co*, *m*, and *n*, are estimated from time-series data, as are the parameters from the discovery rate function, r_o, *u* and s. The parameterization of the Pindyck model is such that the price of oil decreases as supplies increase, for positive values of the parameter *b*, and the average cost of production increases—for positive values for *co* and *n*—as the proven reserve base is depleted, while exploratory costs increase with exploration.[17] First-order necessary conditions for the oil firm optimization problem are given in the Appendix (B.2).

Electric Utility

The electrical utility manager chooses the overall level of electricity generation, and levels of capacity utilization and natural gas and labor inputs to minimize costs, subject to technology and supply. We specify the optimization problem of the utility agent as in equation (13):

$$\min_{\{E_u, NG_u, L_u, CAPU_u\}} \quad p^g \cdot NG_u + w_f \cdot L_u + ucc \cdot CAPU_u \cdot K_u \tag{13}$$

s.t.

$$E_u^{ng} = \psi_u \cdot NG_u^{\partial 1u} \cdot L_u^{\partial 2u} \cdot \left(CAPU_u \cdot K_u\right)^{\partial 3u} \tag{14}$$

$$E_f + E_h + E_g \leq E_u^{ng} + \overline{E}_u^h \tag{15}$$

where the utility manager operates thermal and hydro-electricity plants. She chooses L_u and NG_u, the levels of labor and natural gas inputs to use, and E_u, the amount of electricity produced. She also determines the levels of utilization of installed capacity

[17]Production costs increase as the proven reserve base is depleted to represent the increasing marginal costs of making new discoveries over time.

($CAPU_u$), while *ucc* and K_o are exogenously determined levels of user cost and investments in electric power capacity, respectively. Output of electricity in the thermal plants, E_u^{ng} is represented by the Cobb-Douglas technology function defined in equation (3.13) so that $\partial 1k$, $\partial 2k$ and $\partial 3k$, the relevant production coefficients on natural gas, labor and capital inputs, sum to one. Additional electricity from hydro-resource units, $\overline{E}_u^h$, is assumed to be a fixed proportion of electricity produced in units using thermal energy.[18] A final technical restriction on the utility's optimization problem is that the total grid electricity consumed by the public and private consumers does not exceed electricity produced by thermal- and hydro- stations. E_f, E_h and E_g in equation (15) represent electricity consumption by the government, residences and private firms. We present the first order conditions for the electric utility in the Appendix (B.3).

Households

In this model, residents are assumed to own all local and foreign capital used in production by firms. The household agent chooses levels of her consumption of energy and other goods and her levels of capital formation. The household objective is specified as an iso-elastic intertemporal utility function as in equation (16).

$$\underset{\{C, E_h, I_f, MGS\}}{MAX} \int_{t0}^{\infty} e^{-\beta ht} \left\{ \frac{1}{\gamma} \left(C \cdot MGS^{\eta 1} \cdot E_h^{\eta 2} \right)^{\gamma} \right\} dt, \ \eta 1, \eta 2 > 0, \ -\infty < \gamma < 1.0. \qquad (6)$$

s.t.

$$\dot{nfa} = \phi_h(orev) + (1-\tau_y)(r.nfa + Y) - (1-\tau_c)(C + MGS) - I_f(1 + \frac{\nu I_f}{2K_f})$$
$$- E_h \cdot P_e \qquad (17)$$

$$\dot{K}_f = I_f \qquad (18)$$

18 Huge capital outlays for expansion projects and natural constraints on input availability make the year-to-year responses to energy demand more difficult to model in hydroelectricity facilities.

where the consumer chooses *C*, her aggregate level of goods consumed and E_h, her total household energy consumption in value terms. She also chooses I_f, her levels of investment in local private (non-oil) firms. The constraint on household spending is specified in equation (17). *nfa* denotes net foreign assets and is in form of foreign bonds held. τ_y and τ_c are taxes imposed on income and consumption while *r* denotes the rate of returns on foreign capital. The household agent receives a direct public fund allocation or income transfers in form of a fixed percentage (ϕ_h) of government revenues from the oil companies, *orev*. *Y* is gross domestic production and is determined in the model as firms' output so that it enters into the household optimization problem as an exogenous variable; *MGS* is the aggregate value of imports of goods and services, chosen by the households. As in the government agent optimization problem, quadratic costs of adjustments are imposed on changes to levels of capital owned by households—i.e. $(1+\nu_1 I_f/2K_f)$ in equation 3.17. Equation (18) denotes the physical constraint on the formation of household capital in firms. The relevant first-order conditions for optimization are presented in appendix (B.4).

Local Private Firms

The representative agent for firms chooses levels of capacity utilization and energy and labor inputs to maximize net revenues, subject to expected output demand and technology constraints. The optimization problem of the firm is presented in Equations (19) through (21).

$$\underset{\{E_f, L_f, CAPU_f\}}{MAX} P^{gdp} \cdot Y_f - \tau_{gg} \cdot GG_f - W_f \cdot L_f - P^e \cdot E_f - ucc \cdot CAPU_f \cdot K_f \qquad (19)$$

s.t.

$$Y_f = \psi_f e^{\lambda_f t} (CAPU_f \cdot K_f)^{\delta_{1f}} L_f^{\delta_{2f}} K_{anr}^{\delta_{3f}} E_f^{\delta_{4f}} \qquad (20)$$

$$GG_f = \psi_{gg} e^{-\lambda gg} \cdot E_f^{\delta_{gg}} \qquad (21)$$

where L_f is the firm's total demand for labor; E_f is energy consumed by the firm and $CAPU_f$ is the level of utilization of installed capacity, K_f. Net revenue for the firm is the value of its total production less the costs for labor, energy and capital inputs. W_f denotes the unit wage rate; P_e is the price paid per unit of electricity consumed in production and *ucc* is the user cost of capital. The firm's output, Y_f, is produced following a Cobb-Douglas production function with energy, labor and capital as inputs. Further, taxes, τ_{gg} may be paid to the government on taxable levels of greenhouse gas emissions, GG_f. Finally, the level of greenhouse gas emissions resulting from the firm's production could be modeled as a function of the level of energy used as in equation (21). A negative value for λ_{gg}, the exponential factor in the greenhouse gas emissions equation could indicate, for example, a decrease in the ratios of emissions from energy inputs over time as 'cleaner' production technologies, are developed.[19] The relevant first-order conditions for optimization are presented in appendix (B.5).

Estimation of the Model

To employ the model specified above in simulation exercises, its parameters must be calibrated. Estimates of the parameters can be obtained by following Wymer's approach to estimating continuous-time models of intertemporally optimizing agents from discrete observations (Wymer, 1993, 1997; Donaghy and Wymer, 2011). According to this approach, a full-information maximum likelihood technique is employed, enabling algebraic restrictions on the coefficients of the model to be directly imposed during estimation. Once parameter estimates have been obtained, the qualitative properties of the model can be examined and alternative policy experiments conducted. This approach has been applied in other empirical settings (e.g. bi-lateral trade in Donaghy *et. al.*, 1999 and environmental policy in Balta-Ozkan, *et. al.*, 2007).

[19] The emissions tax function is only included in this specification to demonstrate the possibility for testing a government policy on emissions levels. Throughout, we assume zero taxes on greenhouse gas emissions in our model estimation and simulation exercises, ie., $\tau_{gg} \cdot GG_f = 0$.

An important feature of this approach is that the intertemporal optimization assumption of the representative agent is directly incorporated in the estimation algorithm (Wymer, 1997). The state and co-state equations characterizing a macro-dynamic equilibrium solution are solved such that relevant initial-value and transversality conditions are met that account for the effects of changes in resource endowments on the objective functional at every data point. As in Donaghy, *et. al*. (1999) and Balta-Ozkan, *et al*. (2007), estimation of the macro-dynamic model can directly incorporate the assumptions made of the representative agents into a three-step recursive solution procedure. In the first step, a set of 'observations' is generated on the unobserved variables and transversality conditions. For the first iteration, this generation of observations may be done by incorporating a set of plausible values of parameters – e.g., from theory – into numerical simulations. In the second-step, a variable-step, variable-order Adams method is used to solve intertemporal optimization problem characterized by the equilibrium conditions for each of the data points, given reasonable assumptions on parameter estimates and on the initial values of state and control variables. Relevant boundary point conditions are imposed on the state and co-state (unobserved) variables. The solution of the model must converge at every observation. The solutions for the unobserved variables are updated at every data point. For the variables for which historical series are available, the dynamic solution values are compared with the observed values. A variance-covariance matrix is formed (from the computed residuals). In the third-step, parameter estimates are then chosen to minimize the natural logarithm of the variance-covariance matrix by a quasi-Newton method. Parameter estimates employed in the first step are then updated. The solutions to the model for the specified time horizon are checked for convergence. Steps one through three are repeated if relevant convergence criteria are not met.

Application to the Empirical Setting

We have not yet been able to implement estimation of the full model as outlined above, but we have been able to estimate blocs of equations corresponding to the different representative agents. Blocs of equations were estimated with continuous-time methods with annual country-level data from 1980 to 2006. This length of the series represents the years for which we could obtain reasonably complete data to account for the representative agents' optimization problems specified for the macro-dynamic system. The time series that include data on income, production, consumption, investment and energy were obtained from publications of the Energy Commission of Nigeria and from various independent local and international data sources.[20] Table 2 presents a summary of selected series in our sample of 26 annual observations.

The selected period saw an increase in annual crude oil production from an average of about 540 million barrels in the 1980s to more than 800 million barrels on average, in the last decade for which data were available. As shown in Table 2, oil production peaked at 960 million barrels a year (in 2005). Crude petroleum revenues dominated all payments accruing to the government throughout, ranging from 64 to 86 percent of annual government income. Further, crude oil accounted on average for 95 percent of the value of all exports through the years (not shown in Table). The country's gross domestic production increased 206 percent in real value terms over the period of our sample series while the increase in installed capacity for electricity generation was more modest (170 percent). Total annual electricity consumption by the residences was 243 megawatt hours in 1980 and up to 1,195 megawatt hours by 2005, an increase in energy use over time that may depict growth in such factors as population, income, installed capacity and generation of electricity. Electricity consumption by firms similarly went up in the selected years. We find in the data that the capacity utilization in firms dropped from more than 70 percent in the early 1980s to about 44 percent on average in the last ten years, and 29 percent in 1995.[21]

[20] See Appendix C for a list of the data series as well as their descriptions, sources and modifications employed for the purpose of this research.

[21] Adenikinju (2003) reports that 61 percent of firm respondents in a 1998 nationwide survey estimated that power outages led to drops in their capacity utilization, of 10 to 50 percent.

Time series of currency values were standardized to year 2000 values using the GDP deflator.[22] The periods for which data were available also may have been marked significantly with external and internal shocks that did not lend it readily to formal representation. For example, year-to-year inflation rates of up to 74 percent in the mid-1990s drove close to zero, the real values for some important variables such as wages received in the firm and utility sectors. As such, the potential for characterizing input demand in these sectors may have been seriously compromised.

Initial Model Experimentation and Simulation

To address issues of data unavailability and inaccuracy in the current modeling exercise, necessary simplifications were made to the original specification of the model. The most important of which was that the level of government recurrent expenditures less repayments of debt was defined exogenously (i.e., *ODg* in the set of government equations) as in equation 22.

$$OD_g = \alpha_g \exp(\lambda_{odg} t) \qquad (22)$$

where α_g is a given starting value for government recurrent spending, *ODg*, that represents government recurrent expenditures (less debt repayment) in period (t = 0); and λ_{odg} is the assumed (average) growth rate of government spending over the relevant period, t. All other variables and parameters in the government bloc remain as in equations (2) – (9). We present the relevant parameters obtained from estimating the oil firm, utility and firm agents' modules in Table 3. Variables occurring as non-endogenous (i.e., variables other than the control) in any agent's optimization problem were modeled as forcing functions of time.[23]

[22] The TRANSF program in Wymer's suite of mathematical programs was used to prepare data for estimation.
[23] Forcing functions of time were estimated outside of the structural modules so that the resulting coefficients were independent of the dynamic model parameters (Balta-Ozkhan et al., 2007). The forcing functions for the exogenous variables were estimated using the non-linear FIML estimation program ASIMUL and the blocs of equations corresponding to the different representative agents were estimated in ESCONA.

Table 2: Summary statistics for selected Nigeria data series, 1980 – 2005[1]

Series	Unit and Scale[2]	Minimum	Maximum	Median	Mean	Standard Deviation
Gross Domestic Product	Billions, Naira	4,815.14	14,735.30	7,161.27	7,685.15	2,701.57
Government Recurrent Expenditure	Billions, Naira	184.36	1,117.73	456.75	558.69	257.95
Household Expenditure	Billions, Naira	3,415.93	8,716.88	4,265.13	4,957.76	1,666.61
Government Capital Expenditure:						
Agriculture and Other Non-Oil	Billions, Naira	5.68	92.24	20.50	34.17	28.40
Education and Social Services	Billions, Naira	17.99	302.55	51.28	64.07	56.79
Energy and Utilities	Billions, Naira	8.46	802.28	149.09	212.96	212.22
Crude Oil Production	Millions, Barrels	452.97	959.02	712.30	682.22	136.23
Revenues from Oil Production	Billions, Naira	599.21	4,762.40	1,165.34	1,623.47	1,084.03
Revenues from Non-Oil & Taxes	Billions, Naira	225.90	918.94	395.06	460.19	198.56
Electricity:						
Installed Capacity of Utilities	MegaWatts	2,419.90	6,538.30	6,268.30	5,438.55	1,302.26
Generation	MegaWatt Hours	815.10	2,779.30	1,672.85	1,620.13	521.40
Consumption (Households)	MegaWatt Hours	194.17	1,195.13	490.66	526.70	216.11
Consumption (Firms)	MegaWatt Hours	120.91	398.23	236.39	235.58	52.00
Capacity Utilization of Firms	Percent	29.30	73.30	40.35	42.94	11.25

[1] Various sources. See References for Appendix 3.

[2] Money values deflated using gross domestic product deflator. 2005 = 1.

Table 3: Parameters of the modules for the Intertemporally Optimizing Agent Behaviors[1]

Parameter	Value	Parameter	Value
Government Agent:		*Oil Firm Agent*:	
$a1$	8.00	a	2.688
$a2$	20.00	b	1.655
$a3$	100.00	co	3.595
$\alpha 1$	0.75	m	-0.005
$\alpha 2$	0.25	ro	0.363
βg	0.03	s	0.500
$\lambda 1t$	0.02	u	0.003
$\lambda 2t$	0.02	n	0.0087
$\lambda 3t$	0.02	βo	0.018
λODg	0.03		
$\varpi g1$	1.00	*Private Firm Agent:*	
$\varpi g2$	1.00	$\delta 1f$	0.56
$\varpi g3$	1.00	$\delta 2f$	0.30
ϕg	0.96	$\delta 3f$	0.10
τc	0.06	$\delta 4f$	0.04
τy	0.12	ψf	2.16
γw	1.1	λf	0.02
Household Agent:		*Energy Utility Agent*:	
βh	0.03	$\delta 1u$	0.16
$\eta 1$	0.22	$\delta 2u$	0.33
$\eta 2$	0.09	$\delta 3u$	0.51
ϕh	0.04	ψu	1.80
ν	10.00	λu	0.02
γ	-0.10		

Model Simulation Exercise

The parameter values in Table 3 were obtained either from estimations of the independent agent blocs using available data (i.e., for the oil, local non-oil and energy utility firm modules) or deduced from the literature. These parameter values and given values of control and state variables were used to structure the simulation exercise with the full model. The calibrated zero- and first-order conditions of the optimization problems for the five agents were solved subject to boundary conditions over a relevant time horizon as a nonlinear system of simultaneous equations (see Appendix B). The time-path solutions of the variables indicate how the economy might evolve, given the interrelationships between the agents. It is important to emphasize here that the solution of the current simulation exercise only represents possible future outcomes, given the model capturing of the historical trend. The values are not a prediction of future economic conditions.

Following the evidence from the available data, the tax rates on consumption and income (i.e., τc and τy) were set at 6 and 12 percent, respectively. The assumed values for the adjustment costs on investment variables (i.e., ν, *a1*, *a2* and *a3*) were modified for the current exercise from upper bounds used in Barro and Sala-i-Martin (1995). Following the historical trend, real interest rate in the economy was pegged at 3 percent over the period for which the economy was simulated. The parameters on the proportions of government oil revenues retained by the government or allocated to residents (i.e., ϕg *and* ϕh, respectively) denote a baseline agreement for oil revenue sharing in Nigeria. The parameters for government policy weights on capital accumulation (i.e, $\varpi g1$, $\varpi g2$ and $\varpi g3$) were arbitrarily set to 1.0, denoting equivalent importance for the types of capital. Other plausible values were derived from the data (e.g., $\eta 1$ and $\eta 2$) and from expert opinion (e.g., βg and βh). Starting values for the unobserved variables in the model are presented in Table 4.

Table 4: Values for Unobserved Model Variables

Variable	Starting values defined for t = 0	t = 60
λ	-0.001	Undefined
ϑ	-7.1	Undefined
$\mu 1$	-0.02	0.0
$\mu 2$	0.18	0.0
ζ	-0.0003	Undefined
qganr	10.49	Undefined
qgsc	3.0	Undefined
qgo	22.8	Undefined

Next, we present in Table 5, the initial values for observed variables in the intertemporal optimization model. This set of values represents the values observed for a single observation of data. Typically, a data point (i.e., time, *t*) is chosen that is a viable candidate for a long-term equilibrium. This could be a period in the series for which important variables are somewhat stable from year to year. For our modeling purposes, starting values for the observed variables were chosen for year 1995 of the 26 years (1980 – 2005) of available time series data. ***Results from the Initial Model***

Experimentation

The simplified model of the intertemporally optimizing agents was solved for a time horizon of 60 years from the initial period ($t = 0$). This length of time is considerable, given the level of complexity of agent and variable interactions in the model, and is consistent with the long-term window typically adopted for oil resource and development planning.[24]

[24] However, the initial root mean squared error (0.206325) indicates room for improvement in the model fit to the generated data.

Table 5: Initial Values of Observed Model Variables

Variable	Description (Unit)[1]	Value at t = 0
B	Government debt	1.411
C	Household consumption	2.548
Ianr	Change in capital stock in agriculture	-0.900
If	Change in capital stock in firms	0.100
Io	Change in capital stock in utilities	0.200
Isc	Change in human capital investment	0.100
Kanr	Capital in agriculture	0.665
Kf	Capital in firms	6.816
Ko	Capital in utilities	2.938
Ksc	Capital in human development	0.889
MGS	Consumption of foreign goods	0.500
Nfa	Net foreign assets	0.071
ODg	Government recurrent expenditure	0.200
P	Price of oil (*Naira*)[2]	1.600
Q	Quantity of oil produced	0.700
R	Proven reserves of oil	17.900
W	Number of oil wells drilled	7.200
X	Cumulative additions of reserves	26.789
Y	Economy output	3.000
Ypf	Output from local nonoil firms	3.000

[1] Variable units are as reported in Table A.3 in the appendix; and are scaled as reported here for standardization of units in the model.

[2] Assuming 1995 real price and constant rice of USD 80 per barrel.

Relevant variables were relatively stable over the simulated period with immediate results indicating high importance of the household and government budget equations, both of which were defined as linear identities of the agent's incomes and expenditures. The model's solution provides some information on such policy questions as the optimal rates of oil extraction and exploration in Nigeria. The results are for a baseline measure of direct oil wealth transfers to households and track investments in other (non-oil) sectors.

A full range of sensitivity analyses was not conducted. However, the current specification of optimality conditions depicting relevant agent interactions was found to respond significantly to changes in adjustment costs on investments. The solution paths of key variables are summarized below.

Optimal Oil Extraction and Exploration Rates

The model's determination of optimal production and drilling rates for the oil resource in Nigeria are presented in Figure 2.

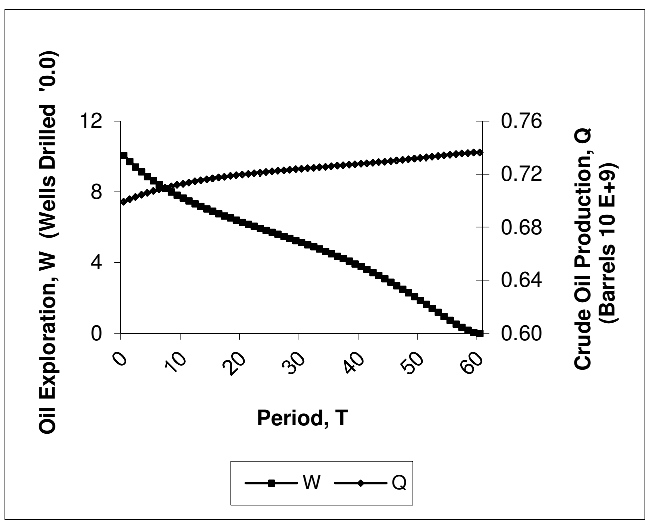


Figure 2: 60-Year Simulation for Optimal Crude Oil Production and Exploration

Based on the properties of the Nigerian economy as represented by the simplified optimization model, the year-to-year increase is expected to dampen over the simulation period while oil production increases. In particular, crude petroleum production increases every year by about 0.25 percent in the initial years of the simulation but this rate tapers to 0.05 percent on average in the last decade of the simulations (i.e. from years 50 through 60). In level terms, crude oil production starts out at over 699 million barrels annually (approximately 1,916 thousand barrels per day) in the initial time and rises roughly 5 percent over the forecast period, to about 2,018 thousand barrels a day by end of period. New oil drilling on the other hand is expected to fall rapidly. The results suggest that oil exploration would fall from a peak drilling rate at the start of the forecasts of around 100 new wells a year, to a new well being drilled every other year, in the terminal simulation period.

Crude petroleum prices move in the opposite direction of production so that the real price of oil decreases as an exponential function of time as output levels increase. Oil revenues in addition are sensitive to production and rise with increasing production (see Figure 3).

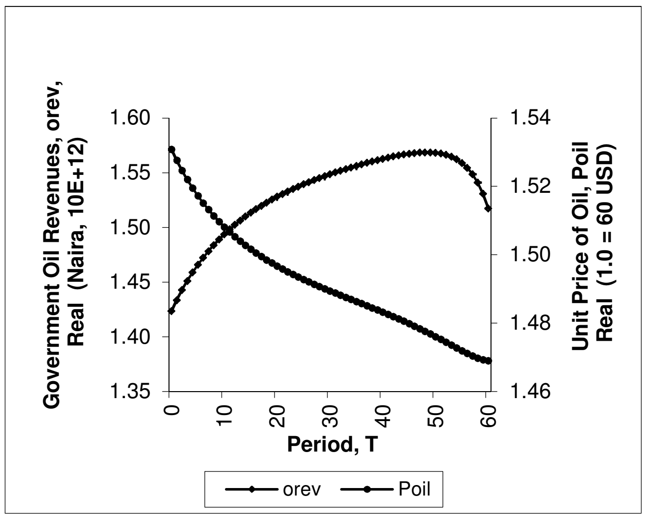


Figure 3: 60-Year Simulation of Revenues and Real Prices of Oil

However, price reductions become sufficiently large as to counter the rise in income (following increased production), so that oil revenues dip at the end of the forecast period. The projected movements in oil prices and revenues are depicted in Figure 3.3. Oil revenues rise by up to 10.2 percent (from about 1,420 million local currency in real value) to the period (t = 48), after which revenues fall by up to 3.3 percent by the end of the forecast period. Oil prices on the other hand record a four percent increase through the period.

Output in Economy

From the solutions to the optimization model of the Nigerian economy, economic output increases substantially over the simulation period (see Figure 4). Output follows an increasing growth pattern almost throughout with an average growth rate of about 4.5 percent per year.[25]

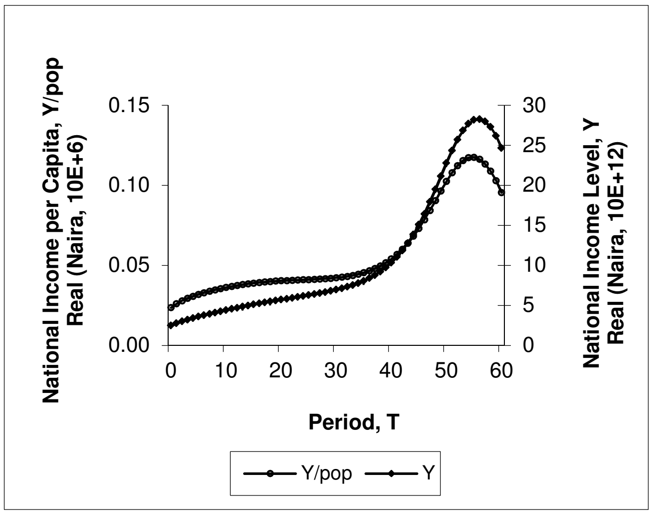


Figure 4: 60-Year Simulation of Level and Per Capita Output of Nigeria Economy

The intertemporal model solutions suggest an almost ten-fold increase in output by the end of the period. When compared with the expected growth in population, the increased output seems to pass on to improvements in the welfare of residents. An annual growth in population of 1.5 percent (based on United Nations projections) is accompanied by per-capita output increases of up to 2.4 percent per year on average over the simulations. However, the income per head falls by an average of 4 percent from year-to-year for the last five years of the simulations. This is consistent with the decreases in output observed for the overall output levels over the same period (as in Figure 4).

[25] Real GDP levels rise by 4.5 percent on average for over 5 decades, then drop by about 3 percent per annum on average over the last four years of simulations.

Investment by the Government

In addition to the oil industry and general economy, important results come out of the government agent model. The levels of government investment in agriculture and human capital development are presented in Figure 5.

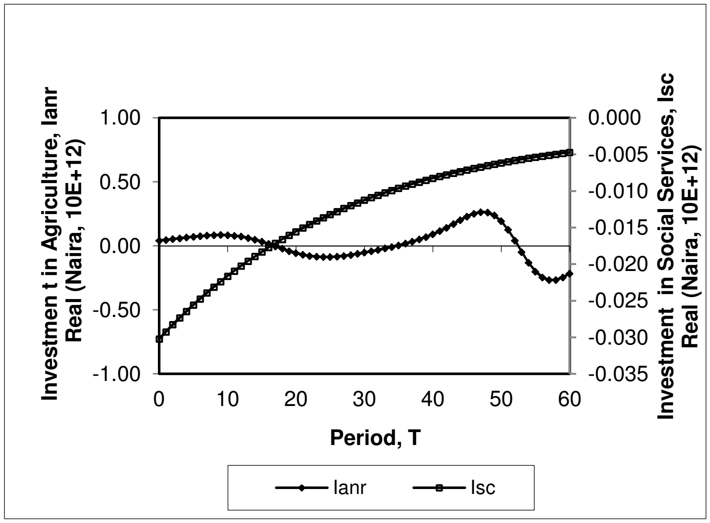


Figure 5: 60-Year Simulation of Investment in Agriculture and Human Capital

The level of capital spending in social and community services increases throughout, but at a decreasing rate, i.e., as more and more social capital is accumulated, the rates of new investments in this form of development are reduced. On the other hand, investments in agriculture and natural resources are almost level for a period, falling significantly several years into the future. This could suggest that, given the starting assumptions of the current model, increased government investment in the agricultural sector may not provide optimal use of limited public resources. More detailed modeling of the economy is however required to reach a more substantive conclusion on the prospects (or constraints) of direct transfer of public funds into specific sectors of the economy to boost market-led economic growth.

The model solutions in addition provide information on the government agent investment in the public energy utility. The level of government investment in energy utilities is shown in Figure 6. According to our model solutions, investments in electricity production increase following an exponential function for more than a quarter of the simulation period, then declines at a more significant rate.[26] The energy investment growth rates may not be inconsistent with huge lump investment patterns traditionally observed in the energy industry, particularly in developing regions. The production of electricity associated with the levels of government investment in the industry as shown in Figure 6, is represented in Figure 7 below.

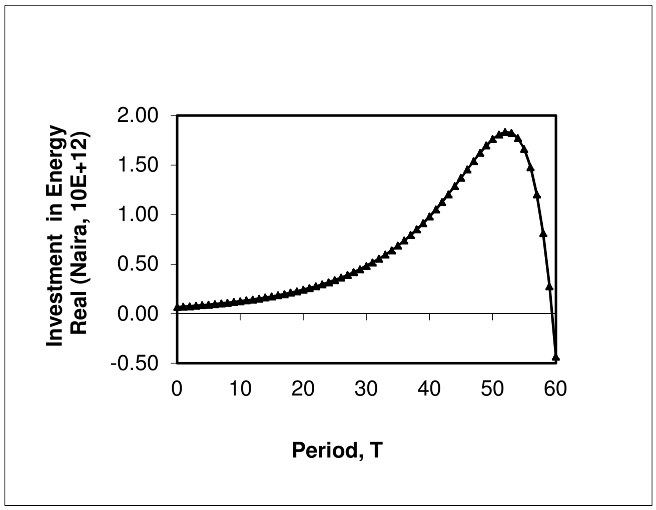


Figure 6: 60-Year Simulation of Investment in Electrical Utilities:

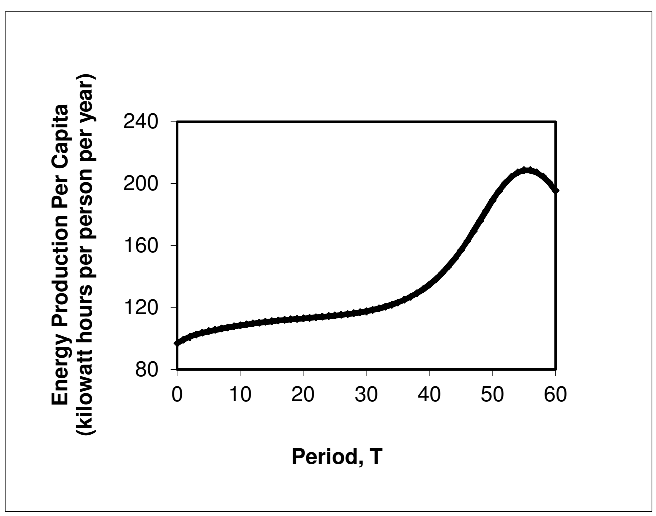


Figure 7: 60-Year Simulation of Electricity Production Per Capita

[26]The late decline is most likely an artifact of the transversality condition on the capital stock of the energy utility.

Electricity production and consumption (not shown in figure) increase over time. While production declines at the end of the simulation period, it is still significantly higher than the unit level of electricity produced in the initial year of the energy projections. This solution might underscore the need for improved electricity supply to support national economic development.

Household Consumption and Investment

Households in the optimization model respond to the improved levels of output with increased spending on goods (see Figure 8). The year-to-year adjustments indicate monotonic changes that cause consumption to rise by about 32 percent through the end of the projections. Expenditures on local goods and services (including energy) increase at a faster rate than consumption of imports.

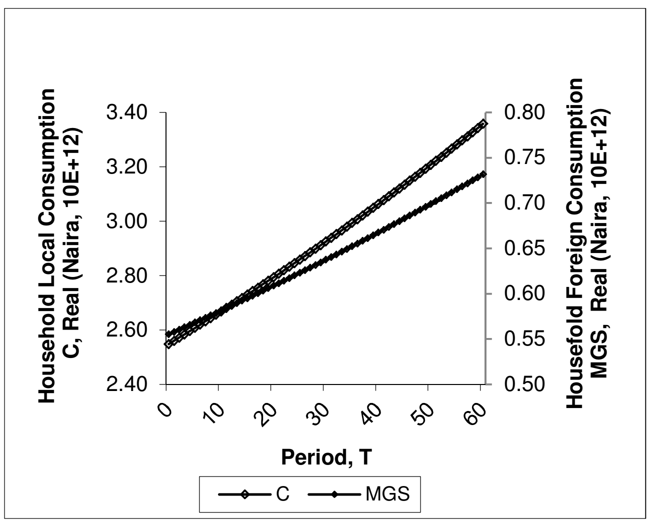


Figure 8: Households Consumption of Goods: 60-Year Simulations

Further, according to the model output, households increase their investments in local private firms. The investments grow from a period of no new investments, to a relatively lengthy period of stable investments in local industry and finally, to positive and increasing capital spending at the end of the model projections. The model solutions thus indicate increased transfers of household wealth into firm production, consistent with increased production and consumption of locally- made goods in the economy (Figure 8).

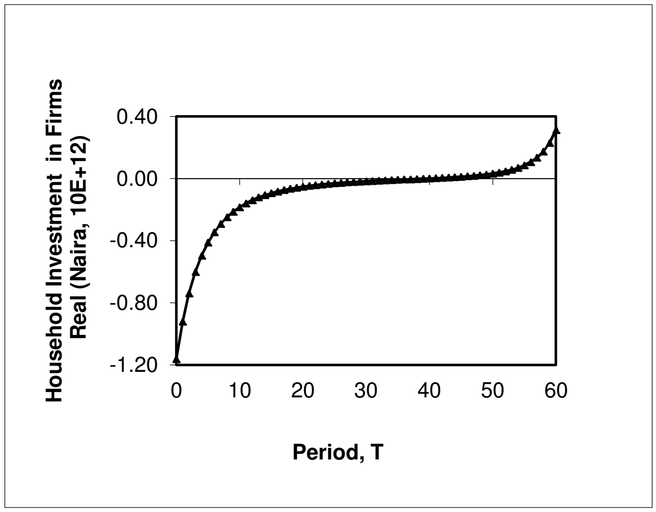


Figure 9: Household Investment in Firms: 60-Year Simulations

The solution paths of these variables further suggest that much of the increase in national income is transferred into ownership of foreign assets. (See Figure 10)

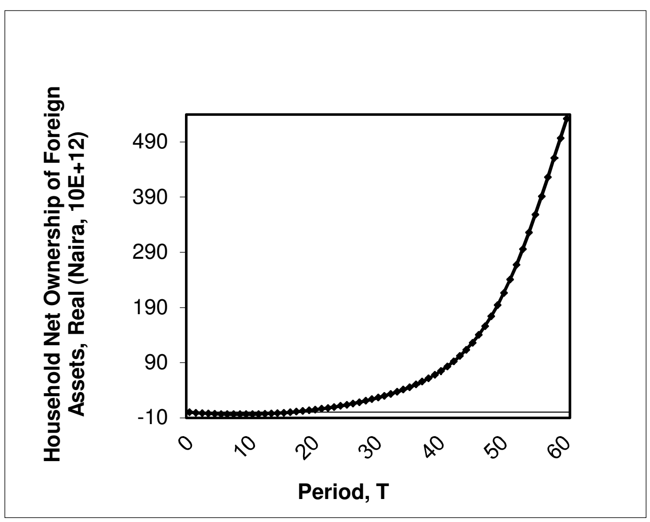


Figure 3.10: 60-Year Simulation of Net Foreign Asset Ownership

Summary of Results and Implications

The current research set out to apply recent developments in macro-dynamic modeling techniques to an energy-economy planning problem in Nigeria. The specification of the model of the economy accounted for the dependence of the economy on crude petroleum, a nonrenewable natural resource, and outlined intertemporal optimization objectives of distinct economic agents. The model specification allowed for the transfer of the wealth accrued from oil exports into capital expenditures on three major types of investments representing the government objectives for economic diversification (or the development of an alternative industry to oil export earnings); human capital; and local energy development. It also allowed for households to make investments in a local non-oil production or manufacturing sector and accounted for activities in a publicly-held energy utility. Components of the intertemporal optimization model were estimated for each of the economic agents to obtain model parameter values for a forecasting exercise. Plausible estimates were obtained for other parameters and unobserved variables from the literature. In addition, initial values of the observed variables were obtained from available data series using regression and similar techniques. Solutions to the model provided indicators on the optimal production and exploration rates for crude petroleum in Nigeria that are useful from a planning perspective. The solutions also suggest important information for investment in the various types of capital useful for socio-economic development.

Oil production increased over a 60-year planning horizon although year-to-year increases in production were observed to fall slowly. Optimal oil exploration rates declined at a much faster rate, decreasing to almost no new wells being developed at the end of the forecast period. Projections of oil prices showed expected declines in the future as optimal production increased. On the other hand, national (government) revenues from oil are expected to increase on average in association with the increased supply of the crude oil output, although declining prices at the end of the forecast period overshadow the effects of increased production so that total revenues from oil rents eventually decline. The results could indicate that the Nigerian economy is fast approaching a peak crude production level beyond which potential production could decline, given the reserves. Potential for reduced incomes from oil rents, in addition to further risks from external price shocks may state the case for increased diversification of the resource and economic base in Nigeria. However, the results indicate that whilst it is optimal for the government to increase its levels of investments in human capital and energy development, increased direct involvement in agriculture and natural resources is not optimal. In particular, the model projections show level investments in agriculture and natural resources for a relatively lengthy period and significant declines into the future.

In the production sector, model projections show increased output and income in the economy on absolute and per capita basis. Households also show increased consumption of both local and foreign goods and services. National ownership of foreign assets is also shown to increase. While these measures together suggest improved levels of welfare in the economy, it should be noted that the results from the current modeling exercise may not be directly applicable to policy interpretations. The results are however sufficiently interesting to suggest continued development of models of this nature.

Further Development of the Model and Future Research

Immediate changes that could be made to the model to improve the characterization of the Nigeria economy and its application to forecasting and policy simulation include accounting more completely for interactions between and amongst the representative agents. For example, an attempt could be made to re-define the variable representing the government recurrent spending as an endogenous variable so that the direct effects of its changes can be explicitly accounted for in the household objective function. Further, production in the petroleum industry could be modeled to influence natural gas supplies to utilities producing electricity for the economy. Labor demand in households and non-oil firms should in a more complete characterization of the economy correspond to labor supplied by the households and should account for a proportion of the income coming to residents. However, improvements to the model need be accompanied with improvements in the quality of the data for any meaningful progress to be achieved. In more advanced developments of the current model, the levels of interactions between the oil sector and other economic agents could be specified in increased detail to allow the model more completely simulate the non-autonomous nature of the oil industry (i.e., from government regulation) and the involvement of households and local private firms in the oil industry. Agent representation in the model can be expanded to account for an agricultural sector with for example, formal, informal, crop and livestock sub-sector classifications that allow for the tracking of labor movements between agriculture and other economic sectors.

APPENDIX A
OBJECTIVES FOR NIGERIA NATIONAL ENERGY POLICY

1. Achieve national energy security and an efficient energy delivery system using an optimal mix of the nation's diversified energy resources.
2. Guarantee increased contribution of energy productive activities to national income.
3. Guarantee adequate, reliable and sustainable energy supply at appropriate costs and in an environmentally friendly manner, to the various sectors of the economy.
4. Guarantee an efficient and cost effective consumption pattern of energy resources.
5. Accelerate the process of acquisition and diffusion of technology and managerial expertise in the energy sector and indigenous participation in energy sector industries, for stability and self-reliance.
6. Promote increased investment and development of the energy sector with substantial private sector participation.
7. Ensure comprehensive, integrated and well-informed energy sector plans and programs for effective development.
8. Foster international co-operation in energy trade and projects development in Africa and in the world at large.
9. Promote international co-operation.

Source: ECN, 2003

APPENDIX B

THE FIRST ORDER NECESSARY CONDITIONS

B.1: The Government

In addition to the 3 state equations defined in the text, the other first-order necessary conditions for the government agent are derived as in equations (B.1.1) through (B.1.8). The relevant boundary point and transversality conditions are in equations (B.1.9) through (B.1.12):

$$\partial H / \partial B : \dot{\vartheta} = -\vartheta \{ r \cdot (1 - \tau_y) - \beta_g \} \tag{B.1.1}$$

$$\partial H / \partial K_{anr} : \dot{q}g_{anr} = -\varpi_{g1}\left(K_{anr} - K_{anr_0} e^{\lambda 1 t}\right) - \vartheta\left(\frac{a_{anr}}{2} \cdot \left(\frac{I_{anr}}{K_{anr}}\right)^2\right) + \beta_g . qg_{anr} \tag{B.1.2}$$

$$\partial H / \partial K_{sc} : \dot{q}g_{sc} = -\varpi_{g2}\left(\Omega - \Omega_0 e^{\lambda 2 t}\right) \cdot \gamma w \cdot \frac{\Omega}{K_{sc}} + \vartheta\left(\frac{a_{sc}}{2} \cdot \left(\frac{I_{sc}}{K_{sc}}\right)^2\right) + \beta_g . qg_{sc} \tag{B.1.3}$$

$$\partial H / \partial K_o : \dot{q}g_o = -\varpi_{g3}\left(E^{ng} - E_0^{ng} e^{\lambda 3 t}\right) \cdot \delta 3k \cdot \frac{E^{ng} \cdot Pe}{K_o} + \vartheta\left(\frac{a_o}{2} \cdot \left(\frac{I_o}{K_o}\right)^2\right) + \beta_g . qg_o \tag{B.1.4}$$

$$\partial H / \partial I_{anr} : I_{anr} = -K_{anr} \cdot \left(1.0 + \frac{qg_{anr}}{\vartheta}\right) / a_{anr} \tag{B.1.5}$$

$$\partial H / \partial I_{sc} : I_{sc} = -K_{sc} \cdot \left(1.0 + \frac{qg_{sc}}{\vartheta}\right) / a_{sc} \tag{B.1.6}$$

$$\partial H / \partial I_o : I_o = -K_o \cdot \left(1.0 + \frac{qg_o}{\vartheta}\right) / a_o \tag{B.1.7}$$

$$\partial H / \partial OD_g : \varpi_{g2}\left(\Omega - \Omega_0 e^{\lambda 2 t}\right) \cdot \alpha 2 \cdot \frac{\Omega}{OD_g} + \vartheta = 0 \tag{B.1.8}$$

$$B_{(to)} = B_o ; \tag{B.1.9}$$

$$K_{anr(to)} = K_{anr} ; \quad K_{sc(to)} = K_{sc} ; \quad K_{o(to)} = K_o ; \tag{B.1.10}$$

$$\lim_{to \longrightarrow \infty} \vartheta e^{-Bt} B = 0 ; \quad \lim_{to \longrightarrow \infty} q g_{ag} e^{-Bt} K_{anr} = 0 ; \tag{B.1.11}$$

$$\lim_{to \longrightarrow \infty} q g_{sc} e^{-Bt} K_{sc} = 0 \ ; \quad \lim_{to \longrightarrow \infty} q g_{o} e^{-Bt} K_{o} = 0 \ ; \tag{B.1.12}$$

B.2: The Oil Firm

In addition to the 2 state equations defined in the text, the derived first-order necessary conditions for the oil firm include equations (B.2.1) to (B.2.4):

$$\partial H / \partial X : \dot{\mu}_1 = (\mu_1 + \mu_2) \cdot u \cdot r_o w^s e^{-\mu X} + \beta_o \cdot \mu_1 \tag{B.2.1}$$

$$\partial H / \partial R : \dot{\mu}_2 = -c_o q R^{-2} + \beta_o \cdot \mu_2 \tag{B.2.2}$$

$$\partial H / \partial q : Q = (a \ - c_o / R - \mu_2) / 2 \cdot b \tag{B.2.3}$$

$$\partial H / \partial w : W = \{ s \cdot (\mu_1 + \mu_2) \cdot r_o e^{-u \cdot X} / n \}^{1/s-1} \tag{B.2.4}$$

B.3: The Electric Utility

The first-order necessary conditions for the utility manager (in addition to the constraint equations defined in the text), choosing natural-gas and labor as inputs to minimize the cost of producing some targeted level of capacity of natural-gas based power, E_u^{ng}, are:

$$\partial H / \partial NG_u : NG_u = \partial 1 u \cdot \left(\frac{E^{ng} \cdot P_e}{P_g} \right) \tag{B.3.1}$$

$$\partial H / \partial L_u : L_u = \partial 2 u \cdot \left(\frac{E^{ng} \cdot P_e}{W_u} \right) \tag{B.3.2}$$

$$\partial H / \partial CAPU_u : CAPU_u = \partial 3 u \cdot \left(\frac{E^{ng} \cdot P_e}{ucc \cdot K_o} \right) \tag{B.3.3}$$

B.4: The Household

In addition to the 2 state equations defined in the text, the Hamiltonian system of equations for the representative agent of the household are defined in equations (B.4.1) to (B.4.21):

$$\partial H / \partial nfa : \dot{\lambda} = \beta_h \lambda - (1 - \tau_y) \cdot (\lambda \cdot r) \qquad \text{(B.4.1)}$$

$$\partial H / \partial K_f : -\dot{\xi} = -\beta_h \xi - \lambda \cdot \left[\upsilon / 2 \cdot (I_f / K_f)^2 \right] \qquad \text{(B.4.2)}$$

$$\partial H / \partial C : \dot{C} = C / (\eta_1 \cdot \gamma + \eta_2 \cdot \gamma + \gamma - 1.0) \cdot (\beta_h - r \cdot (1.0 - \tau_y)) \qquad \text{(B.4.3)}$$

$$\partial H / \partial MGS : MGS = \eta_1 \cdot C \qquad \text{(B.4.4)}$$

$$\partial H / \partial E_h : E_h = \eta_2 \cdot C \cdot (1.0 + \tau_c) / P_e \qquad \text{(B.4.5)}$$

$$\partial H / \partial I_f : I_f = -K_f \cdot \left(1 + \frac{\xi}{\lambda} \right) / \upsilon \qquad \text{(B.4.6)}$$

$$\lim_{to \longrightarrow \infty} \lambda e^{-Bt} nfa = 0 ; \quad \lim_{to \longrightarrow \infty} \lambda e^{-Bt} (1 + \nu I_f / K_f) = 0 ; \qquad \text{(B.4.7)}$$

B.5: The Private Local Firm

In addition to the constraint equations defined in the text, the first-order necessary conditions for the firm, choosing labor and energy inputs to minimize the costs of producing gross domestic product, *Y*, are as follows:

$$\partial H / \partial L_f : L_f = \delta_{1f} \left(\frac{Y}{W_f} \right) \qquad \text{(B.5.1)}$$

$$\partial H / \partial E_f : E_f = \delta_{2f} \left(\frac{Y}{P_e} \right) \qquad \text{(B.5.2)}$$

$$\partial H / \partial CAPU_f : CAPU_f = \delta_{3f} \left(\frac{Y}{ucc \cdot K_f} \right) = 0 \qquad \text{(B.5.3)}$$

APPENDIX C

DATA DESCRIPTION

Table A.3: Description of the Nigeria Data Series

Variable	*Source*[1]	*Unit*	*Series Description and Modifications*
arev	CBN	Naira	Gross non-oil revenues
B	CBN	Naira	Sales of government bonds
			Derived: B = (rB / r)
C	EIU	Naira	Household expenditures (private consumption)
Capuf	CBN	Percent	Capacity utilization in firms
Capuu	CBN	Percent	Capacity utilization in utilities
			Derived: percent electricity generated of installed generation capacity
Cg	CBN	Naira	Government recurrent expenditures (government consumption)
Ef	CBN	mWh	Electricity consumption by firms
Eg	CBN	mWh	Electricity consumption by government
			(street lighting and government buildings)
Eh	CBN	mWh	Electricity consumed by residences
Ehu	CBN	mWh	Electricity generation from hydro facilities
Engu	CBN	mWh	Electricity generation from thermal facilities
Eu	CBN	mWh	Total electricity generation
Ianr	CBN	Naira	Capital expenditures on agriculture and natural resources

Table A.3 Continued …			
Ich	I&A	Megawatts	Installed capacity of hydro-electricity
If	IFS	Naira	Capital investments in firms
			(Gross fixed capital formation)
Io	CBN	Naira	Capital expenditures on other economic services
			(transportation, communication, construction and electricity)
Isc	CBN	Naira	Capital expenditures on social and community services
Lf	ILO	Units	Labor employed in firms
			Derived: manufacturing (percent of) GDP*employed persons over age 15
Lu	ILO	Units	Labor employed in public utilities
			Derived: utilities (percent of) GDP * employed persons over age 15
MGS	EIU	Naira	Imports of goods and services
NFA	EIU	Naira	Net foreign assets (commercial banks)
Ngu	NNPC	Cubic feet	Natural gas used in electricity generation (as fixed proportion of gas produced)
orev	CBN	Naira	Gross revenue from crude petroleum
			(international and local sales, petroleum profit tax and treaties)
Pe	CBN	Naira	Unit price of electricity produced
Pgdp	EIU	Unit	GDP deflator (2000 = 1)
Png	CBN	Naira	Unit cost of natural gas used in electricity production
Poil	CBN	Naira	Unit cost of crude petroleum; Derived
r	CBN	Percent	Discount rate

Table A.3. Contd.			
R	EIU	Barrels	Known levels of crude oil reserves
rB	CBN	Naira	Recurrent expenditures on government debt
trn	CBN	Naira	Capital investments on transfers
ucc	IFS	Percent	User cost of capital
	NBS;		Derived: Interest rate plus depreciation less inflation
W	NNPC	Units	Number of oil wells dug
Wf	CBN	Naira	Average annual wage paid in the private sector (manufacturing)
Wu	CBN	Naira	Average annual wage paid in the public sector
X	EIU	Barrels	Cumulative additions of known oil reserves
			Derived: Xt = Xbar + Xt-1, where Xbar = Rt - Rt-1 + Qt for t = year
Xo	EIU	Naira	Value of crude oil exports
Y	EIU	Naira	Gross domestic production, deflated (2000=1)

[1]Sources: CBN, EIA; EIU; I&A: Ibitoye and Adenikinju; IFS; ILO; NBS and NNPC. See References for Appendix 3.

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